Mathematics for Economists (BGSE, doctoral students)

Winter term 2018/19

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Angel de la Fuente: Mathematical Methods and Models for Economists, Cambridge University Press: 2000.

[David Gale: The Theory of Linear Economic Models (Chapter 3), McGraw-Hill Book Company: 1960.]

Outline of the course

0. Introduction: The Kuhn-Tucker Theorem (p. 293)
1. The Real Number System
2. Real Vector Spaces
3. Metric Spaces and Normed Vector Spaces
4. Vector Spaces and Linear Transformations
5. Differential Calculus
6. The Inverse-Function Theorem
7. The Implicit-Function Theorem
8. The Lagrange-Problem
9. The Proof of the Kuhn-Tucker Theorem
10. Applying the Separating Hyperplane Theorem
11. The Separating Hyperplane Theorem
12. Linear Programming

Preparation before attending the class:

It is highly recommended to study chapter 1 (pp. 1-38) of the Fuente book before entering the BGSE

Sections from the book by de la Fuente to be covered in class:

1.5 and 1.6

2.1 – 2.8

3.1 – 3.4

4.1 – 4.4

5.1 and 5.2

6.1 – 6.3

7.1

Section 0: second welfare theorem as an introductory example for the Kuhn-Tucker-Theorem

Section 1:

- Definitions: field; ordered field; complete ordered field
- Results (selection): real numbers as the “only” complete ordered field; supremum property; Archimedean property

Section 2:

- Definitions: real vector space; linear subspaces; $\mathbb{R}^n$; linear mappings; linear independence; basis; dimension; matrix representation of linear mappings; kernel; image
- Results (selection): length of a basis; fundamental relation between dimensions

Section 3:

- Definitions: metric space; limit of an infinite sequence; real numbers as a metric space; system of nested intervals; normed vector spaces; Euclidian vector spaces; E^n; open subsets; closed subsets; limits of functions; continuous functions; Cauchy sequences; complete metric spaces; contraction mappings; compact subsets; sequentially compact subsets
- Results (selection): theorem on nested intervals; Bolzano-Weierstrass; Cauchy-Schwartz inequality; characterization of closed subsets; contraction mapping theorem; Weierstrass; Heine-Borel; many propositions of lesser prominence

Section 4:

- Definitions: continuous linear mappings; bounded linear mappings; norm of a linear mapping; invertible operators
- Results (selection): bounds for the norm of a mapping; set of invertible operators as an open subset of the set of all operators

Section 5:

- Definitions: differentiability of functions from $\mathbb{R}$ to $\mathbb{R}$; partial and directional derivatives; differentiability of functions from E^n to E^m; continuous differentiability
- Results (selection): Rolle; mean value theorem; characterization of continuous differentiability; generalized mean value theorem

Section 6:

- Result: inverse-function theorem

Section 7:

- Result: implicit-function theorem

Section 8:

- Definitions: convex sets; concave, quasi-concave, pseudo-concave functions
- Result: Lagrange theorem

Section 9:

- Definitions: saddle points
- Result: Kuhn-Tucker theorem; sufficient conditions for solutions

Section 10:

- Definitions: Slater condition, boundary of a set
- Result: Concave programming

Section 11:

- Definitions: separating hyperplanes
- Result: Separating hyperplane theorem

Section 12:

- Definitions: Linear programs and their dual problems
- Result: Duality theorems of linear programming